Paper Reference(s)

6672

Edexcel GCE

Pure Mathematics P2

Advanced Subsidiary

Wednesday 19 January 2005 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Lilac)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P2), the paper reference (6672), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. $$f(x) = \frac{x^2 - x - 6}{x^2 - 3x}, \quad x \neq 0, \quad x \neq 3.$$

(*a*) Express f(x) in its simplest form.

(3)

(*b*) Hence, or otherwise, find the exact solutions of f(x) = x + 1.

(3)

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2. **Figure 1**

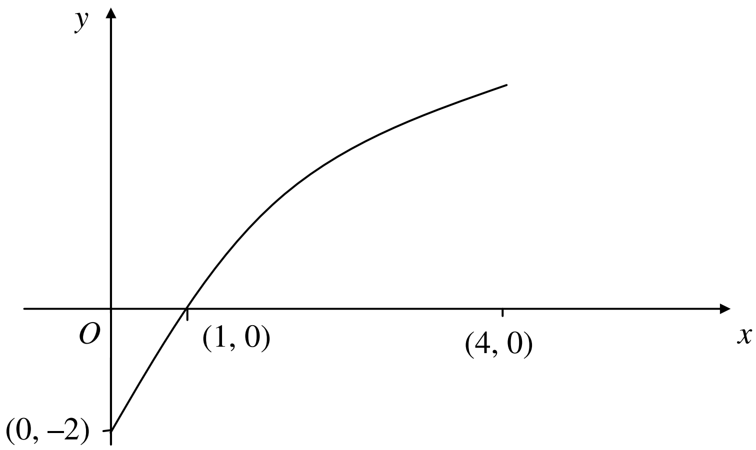

Figure 1 shows a sketch of the curve with equation y = f(x), $0 \leq x \leq 4$. The curve passes through the point (1, 0) on the x-axis and meets the y-axis at the point (0, –2).

Sketch, on separate axes, the graph of

(*a*) $y = |\mathrm{f}(x)|$,

(2)

(*b*) y = f(2x),

(2)

(*c*) $y = \mathrm{f}^{-1}(x)$,

(3)

in each case showing the coordinates of the points at which the graph meets the axes.

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3. The sequence $u_1, u_2, u_3, \ldots$, is defined by the recurrence relation

$$u_{n+1} = (-1)^n u_n + d, \quad u_1 = 2, \quad \text{where } d \text{ is a constant.}$$

(*a*) Show that $u_5 = 2$.

(4)

(*b*) Deduce an expression for u_{10}, in terms of d.

(1)

Given that $u_3 = 3u_2$,

(*c*) find the value of d.

(2)

4.

Figure 2

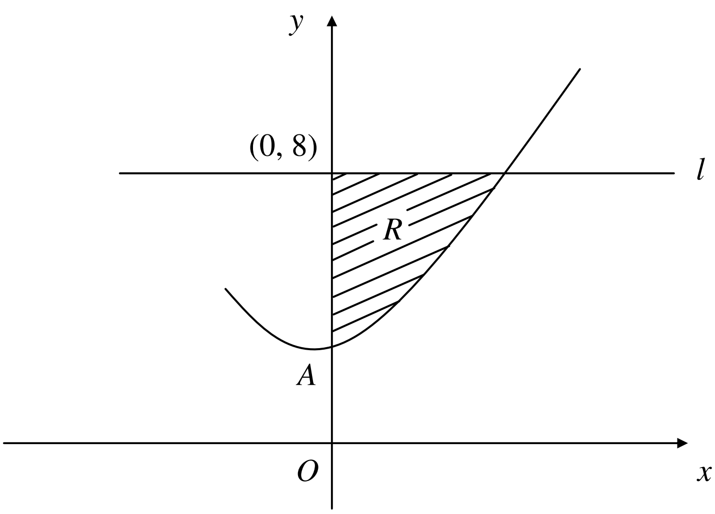

Figure 2 shows part of the curve with equation $y = x^2 + 4$ and the line l with equation $y = 8$.

The curve crosses the y-axis at the point A.

(*a*) Write down the coordinates of A.

(1)

The shaded region R, which is bounded by the curve, the y-axis and the line l, is rotated through 360° *about the y-axis.*

(*b*) Using calculus, calculate the volume of the solid generated, giving your answer in terms of π.

(6)

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5. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(*a*) $3^x = 5$,

(3)

(*b*) $\log_2 (2x + 1) - \log_2 x = 2$,

(4)

(*c*) $\ln \sin x = -\ln \sec x$, in the interval $0 < x < 90°$.

(3)

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6. The function f is defined by

$$\text{f}: x \mapsto 3 + 2\text{e}^x, \quad x \in \mathbb{R}.$$

(*a*) Evaluate $\int_0^1 \text{f}(x) \, \text{d}x$, giving your answer in terms of e.

(3)

The curve C, with equation $y = \text{f}(x)$, passes through the y-axis at the point A. The tangent to C at A meets the x-axis at the point $(c, 0)$.

(*b*) Find the value of c.

(4)

The function g is defined by

$$\text{g}: x \mapsto \frac{5x + 2}{x + 4}, \quad x \in \mathbb{R}, \quad x > -4.$$

(*c*) Find an expression for $\text{g}^{-1}(x)$.

(3)

(*d*) Find gf(0).

(2)

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7. **Figure 3**

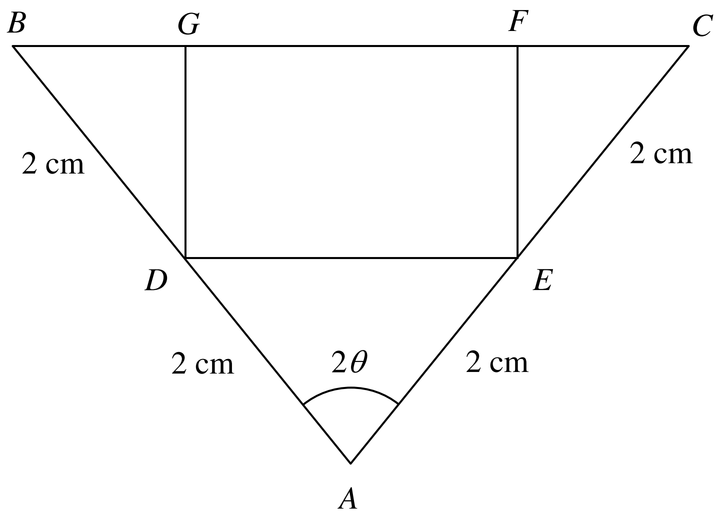


Figure 3 shows an isosceles triangle ABC with $AB = AC = 4$ cm and $\angle BAC = 2\theta$.

The mid-points of AB and AC are D and E respectively. Rectangle $DEFG$ is drawn, with F and G on BC. The perimeter of rectangle $DEFG$ is P cm.

(*a*) Show that $DE = 4 \sin \theta$.

(2)

(*b*) Show that $P = 8 \sin \theta + 4 \cos \theta$.

(2)

(*c*) Express P in the form $R \sin (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

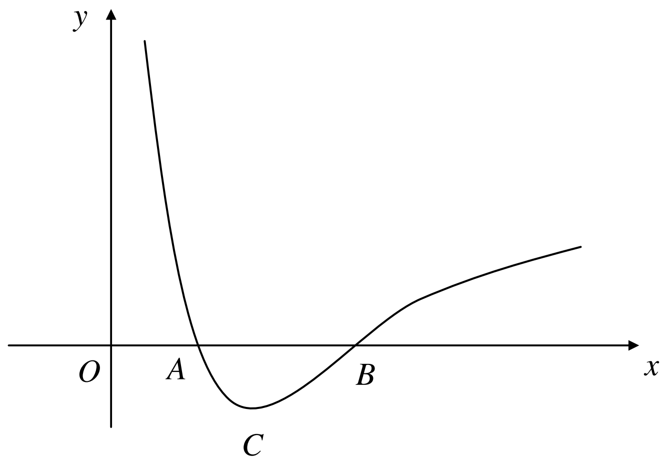
(4)

Given that $P = 8.5$,

(*d*) find, to 3 significant figures, the possible values of θ.

(5)

8. **Figure 4**



$$f(x) = \frac{1}{2x} - 1 + \ln \frac{x}{2}, \quad x > 0.$$

Figure 4 shows part of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points A and B, and has a minimum at the point C.

(*a*) Show that the x-coordinate of C is $\frac{1}{2}$.

(5)

(*b*) Find the y-coordinate of C in the form $k \ln 2$, where k is a constant.

(2)

(*c*) Verify that the x-coordinate of B lies between 4.905 and 4.915.

(2)

(*d*) Show that the equation $\frac{1}{2x} - 1 + \ln \frac{x}{2} = 0$ can be rearranged into the form $x = 2e^{\left(1 - \frac{1}{2x}\right)}$.

(2)

The x-coordinate of B is to be found using the iterative formula

$$x_{n+1} = 2e^{\left(1 - \frac{1}{2x_n}\right)}, \quad \text{with } x_0 = 5.$$

(*e*) Calculate, to 4 decimal places, the values of x_1, x_2 and x_3.

(2)

TOTAL FOR PAPER: 75 MARKS

END